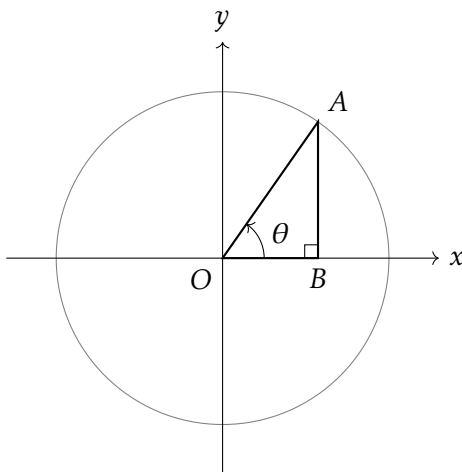


The Unit Circle

Things you should already know

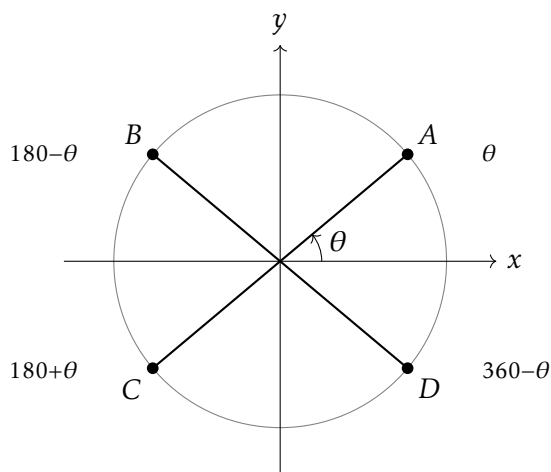
- $\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$, $\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$, $\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$
- SOHCAHTOA works for angles between 0° and 90°
- Right-angled triangle trigonometry



The coordinates of A are $(\cos \theta, \sin \theta)$. The vertical side is $\sin \theta$, the horizontal side is $\cos \theta$, and the radius is 1. This definition works for **all** angles — not just 0° to 90° .

$\tan \theta$ is **defined** as $\frac{\sin \theta}{\cos \theta}$.

The Four Quadrants



Example

In which quadrants is each trig function positive?

<i>Quadrant</i>	<i>Angle range</i>	<i>Positive functions</i>
<i>1st</i>	$0^\circ\text{--}90^\circ$	sin, cos, tan
<i>2nd</i>	$90^\circ\text{--}180^\circ$	sin only
<i>3rd</i>	$180^\circ\text{--}270^\circ$	tan only
<i>4th</i>	$270^\circ\text{--}360^\circ$	cos only

Remember: *All Students Take Coffee (or ASTC)* — reading anticlockwise from the 1st quadrant.

Example

Without a calculator, determine the sign of each:

(a) $\sin 130^\circ$

(b) $\cos 210^\circ$

(c) $\tan 315^\circ$

(a) 130° is in the 2nd quadrant, so $\sin 130^\circ > 0$ (positive)

(b) 210° is in the 3rd quadrant, so $\cos 210^\circ < 0$ (negative)

(c) 315° is in the 4th quadrant, so $\tan 315^\circ < 0$ (negative)

Textbook Exercises: SPSC: 4.10 — Exercise 1A, 1B.

Dr Frost: Trigonometric Graphs slides — unit circle and CAST diagram.

Corbett Maths: Trig Graphs practice questions (Video 362).

Maths Genie: Trigonometric Graphs worksheet.

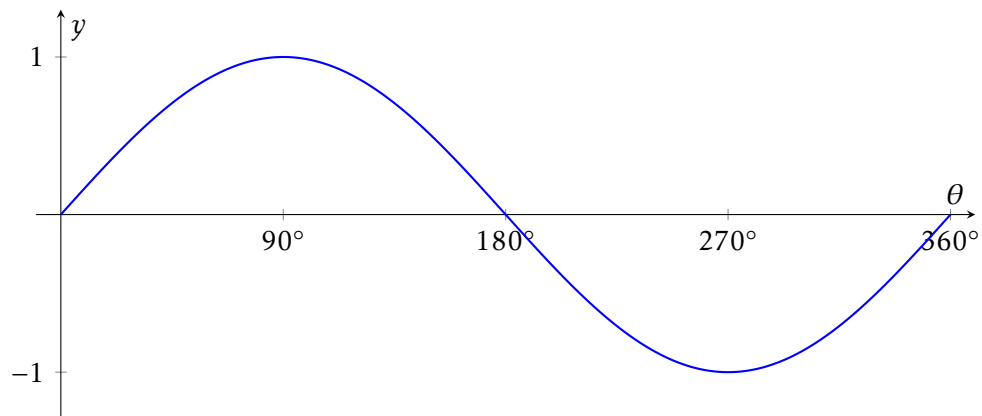
Graphs of the Trigonometric Functions

Example

Using the unit circle, fill in the table for $y = \sin \theta$:

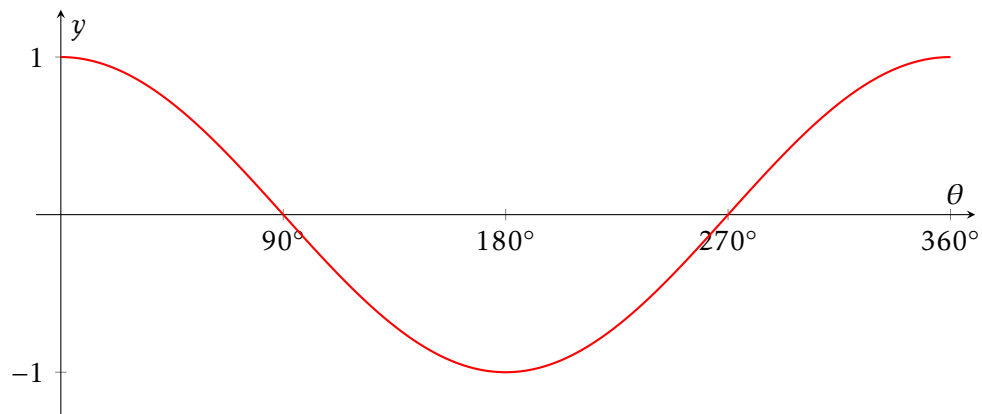
θ	0°	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°	330°	360°
$\sin \theta$													

The graph of $y = \sin \theta$



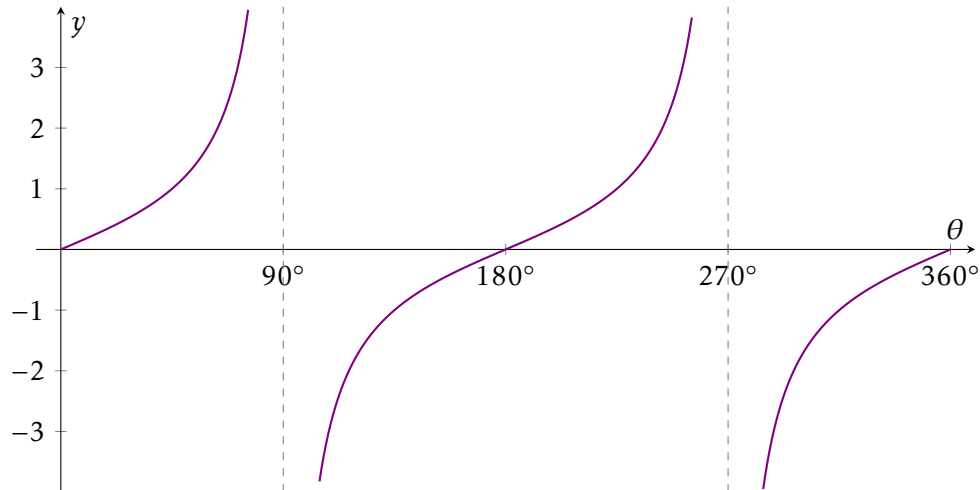
Key properties of $y = \sin \theta$:

- *Maximum value:* 1 (at $\theta = 90^\circ$)
- *Minimum value:* -1 (at $\theta = 270^\circ$)
- *Range:* $-1 \leq \sin \theta \leq 1$
- *Period:* 360°
- *Symmetry:* $\sin \theta = \sin(180^\circ - \theta)$

The graph of $y = \cos \theta$ 

Key properties of $y = \cos \theta$:

- *Maximum value:* 1 (at $\theta = 0^\circ$ and 360°)
- *Minimum value:* -1 (at $\theta = 180^\circ$)
- *Range:* $-1 \leq \cos \theta \leq 1$
- *Period:* 360°
- *Symmetry:* $\cos \theta = \cos(360^\circ - \theta)$

The graph of $y = \tan \theta$ 

Key properties of $y = \tan \theta$:

- $\tan \theta$ is **undefined** at $\theta = 90^\circ$ and 270° (vertical asymptotes)
- Range: $\tan \theta$ can take **any value** (no maximum or minimum)
- Period: 180°
- Symmetry: $\tan \theta = \tan(180^\circ + \theta)$

Example

You are told that $\sin 24^\circ = 0.407$. Without a calculator, write down the values of:

(a) $\sin 156^\circ$

(b) $\sin 204^\circ$

(c) $\sin 336^\circ$

Using the symmetry of the sine graph: $\sin \theta = \sin(180^\circ - \theta)$.

(a) $\sin 156^\circ = \sin(180^\circ - 24^\circ) = \sin 24^\circ = 0.407$

(b) $\sin 204^\circ = \sin(180^\circ + 24^\circ) = -\sin 24^\circ = -0.407$

(c) $\sin 336^\circ = \sin(360^\circ - 24^\circ) = -\sin 24^\circ = -0.407$

Textbook Exercises: SPSC: 4.10 — Exercise 1C.

Dr Frost: Trigonometric Graphs slides — sketching and reading values from graphs.

Corbett Maths: Trig Graphs practice questions.

Maths Genie: Trigonometric Graphs worksheet (symmetry questions).

Solving Trigonometric Equations

Example

Using a sketch of $y = \sin \theta$ for $0^\circ \leq \theta \leq 360^\circ$, how many solutions does $\sin \theta = 0.5$ have? What about $\sin \theta = -0.3$?

Method

Fact — To solve a trigonometric equation over $0^\circ \leq \theta \leq 360^\circ$:

1. Use your calculator to find the **first solution** (the “principal value”).
2. Use the **symmetry of the graph** to find the **second solution**.
3. Check both solutions are in the required range.

Example

Solve $\sin \theta = 0.6$ for $0^\circ \leq \theta \leq 360^\circ$. Give answers to 1 d.p.

$$\theta_1 = \sin^{-1}(0.6) = 36.9^\circ$$

$$\theta_2 = 180^\circ - 36.9^\circ = 143.1^\circ$$

Solutions: $\theta = 36.9^\circ$ or $\theta = 143.1^\circ$

Example

Solve $\cos \theta = -0.4$ for $0^\circ \leq \theta \leq 360^\circ$. Give answers to 1 d.p.

$$\cos^{-1}(-0.4) = 113.6^\circ \quad (\text{calculator gives the value in the 2nd quadrant})$$

$$\theta_1 = 113.6^\circ$$

$$\theta_2 = 360^\circ - 113.6^\circ = 246.4^\circ$$

Solutions: $\theta = 113.6^\circ$ or $\theta = 246.4^\circ$

Example

Solve $\tan \theta = -2$ for $0^\circ \leq \theta \leq 360^\circ$. Give answers to 1 d.p.

$$\tan^{-1}(-2) = -63.4^\circ \quad (\text{calculator gives a negative angle — not in range})$$

$$\text{Add } 180^\circ: \theta_1 = -63.4^\circ + 180^\circ = 116.6^\circ$$

$$\theta_2 = 116.6^\circ + 180^\circ = 296.6^\circ$$

Solutions: $\theta = 116.6^\circ$ or $\theta = 296.6^\circ$

Example

Solve $\sin \theta = -0.5$ for $0^\circ \leq \theta \leq 360^\circ$.

$$\sin^{-1}(-0.5) = -30^\circ \quad (\text{not in range})$$

Sine is negative in the 3rd and 4th quadrants:

$$\theta_1 = 180^\circ - (-30^\circ) = 210^\circ$$

$$\theta_2 = 360^\circ + (-30^\circ) = 330^\circ$$

Solutions: $\theta = 210^\circ$ or $\theta = 330^\circ$

Extension: Equations of the form $\sin k\theta = c$

Fact — For equations like $\sin 2\theta = c$ or $\cos 3\theta = c$:

1. Let $u = k\theta$ and solve $\sin u = c$ in the range $0^\circ \leq u \leq 360k^\circ$
2. Divide all solutions by k to get θ

There are usually $2k$ solutions in any 360° range.

Example

Solve $\cos 2\theta = 0.5$ for $0^\circ \leq \theta \leq 360^\circ$.

Let $u = 2\theta$. We need $\cos u = 0.5$ for $0^\circ \leq u \leq 720^\circ$.

$$u_1 = \cos^{-1}(0.5) = 60^\circ$$

$$u_2 = 360^\circ - 60^\circ = 300^\circ$$

$$u_3 = 60^\circ + 360^\circ = 420^\circ$$

$$u_4 = 300^\circ + 360^\circ = 660^\circ$$

Dividing by 2: $\theta = 30^\circ, 150^\circ, 210^\circ, 330^\circ$

Textbook Exercises: SPSC: 4.10 — Exercise 2A, 2B.

Dr Frost: Trigonometric Graphs slides — solving trig equations section.

Corbett Maths: Solving Trigonometric Equations practice questions (Video 363).

Maths Genie: Solving Trigonometric Equations worksheet.

Maths4Everyone: Trigonometric Equations booklet.